Answers to text book questions

Motion in a plane

Note : Answers given below are intended only as a reference for the questions asked in board examinations. These answers are NOT an attempt at a detailed discussion related to the question.

VSAQs

1. The vertical component of a vector is equal to its horizontal component. What is the angle made by the vector w.r.t. the x-axis?

Let the vector (A) make an angle θ w.r.t. the x-axis

$A_x = A \cos(\theta)$ and $A_y = A \sin(\theta)$

Given $A_x = A_y$ therefore

$A \cos(\theta) = A \sin(\theta)$

$\tan(\theta) = 1$

$\theta = 45°$

2. A vector $\boldsymbol{v}$ makes an angle θ w.r.t. the horizontal. The vector is rotated through an angle θ'. Does this rotation change the vector $\boldsymbol{v}$?

Yes, rotation changes the vector.

A vector has magnitude and direction. When the vector is rotated its direction (and hence its components) changes though its magnitude remains constant.

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3. Two forces of magnitudes 3 units and 5 units act at 60° w.r.t. each other. What is the magnitude of their resultant?

Using the parallelogram law of addition of vectors

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(\theta)}$$

$$F = \sqrt{3^2 + 5^2 + 2\times 3\times 5\times \cos(60)}$$

$$F = \sqrt{49}$$

$$F = 7 \text{ units}$$

4. $\bar{A} = \hat{i} + \hat{j}$ What is the angle between the vector and x-axis?

Given vector is $\bar{A} = \hat{i} + \hat{j}$

Therefore $A_x = 1$ and $A_y = 1$

$\tan(\theta) = A_y / A_x$

$\tan(\theta) = 1$

$\theta = 45°$

5. When two right angled vectors of magnitude 7 units and 24 units combine, what is the magnitude of their resultant?

Using the parallelogram law of addition of vectors

$$R = \sqrt{A^2 + B^2 + 2AB\cos(\theta)}$$
$$R = \sqrt{7^2 + 24^2 + 2\times7\times24\times\cos(90)}$$
$$R = \sqrt{625}$$
$$R = 25 \text{ units}$$

6. If $\bar{P} = 2\hat{i} + 4\hat{j} + 14\hat{k}$ and $\bar{Q} = 4\hat{i} + 4\hat{j} + 10\hat{k}$ find the magnitude of $\boldsymbol{P} + \boldsymbol{Q}$.

Given $\bar{P} = 2\hat{i} + 4\hat{j} + 14\hat{k}$ and $\bar{Q} = 4\hat{i} + 4\hat{j} + 10\hat{k}$

$$\bar{P} + \bar{Q} = (2\hat{i} + 4\hat{j} + 14\hat{k}) + (4\hat{i} + 4\hat{j} + 10\hat{k})$$
$$\bar{P} + \bar{Q} = (2+4)\hat{i} + (4+4)\hat{j} + (14+10)\hat{k}$$
$$\bar{P} + \bar{Q} = 6\hat{i} + 8\hat{j} + 24\hat{k}$$
$$\left|\bar{P} + \bar{Q}\right| = \sqrt{6^2 + 8^2 + 24^2}$$
$$\left|\bar{P} + \bar{Q}\right| = 26$$

7. No question !

8. Can a vector of magnitude zero have non-zero components?

No, a vector with zero magnitude cannot have non-zero components. This is because magnitude is obtained using the sum of squares of components of a vector.

9. What is the acceleration of a projectile at the top of its trajectory?

Acceleration due to gravity (g)
Thought at the highest point, velocity is momentarily zero, acceleration due to gravity causes the descent of the projectile.

10. Can two vectors of unequal magnitude add up to give a zero vector (called a null vector)? Can three unequal vectors add up to give a zero vector?

No, two vectors cannot add up to zero as even for $\theta = 180°$, |A-B| = |A| - |B|
Yes, three unequal vectors may add up to zero. This is possible when the resultant of any two of the vectors is equal and magnitude and opposite in direction to the third vector.

Motion in a plane

SAQs

1. State parallelogram law of vectors. Derive an expression for magnitude and direction of the resultant vector.

   Refer to class notes (or click here for the PDF – page number 12)

   click here for simulation

2. What is relative motion? Explain it.

   Relative motion refers to the motion of a body as observed by an observer located on a stationary/moving frame of reference. All motion that we see around us is relative in nature. There is no absolute motion.
   Example : When a passenger is travelling in a train then he appears to be stationary for a co-passenger in the same train where as for an observer on the ground he appears to me in motion.
   Relative displacement of body A w.r.t. B is given by $\bar{S}_{AB} = \bar{S}_A - \bar{S}_B$
   Differentiating the relation of displacement w.r.t. time we get relative velocity of body A w.r.t. B as $\bar{v}_{AB} = \bar{v}_A - \bar{v}_B$
   Differentiating the relation of relative velocity w.r.t. time we get relative acceleration of body A w.r.t. B as $\bar{a}_{AB} = \bar{a}_A - \bar{a}_B$

3. Show that the boat must move at an angle w.r.t. the river in order to cross the river in minimum time.

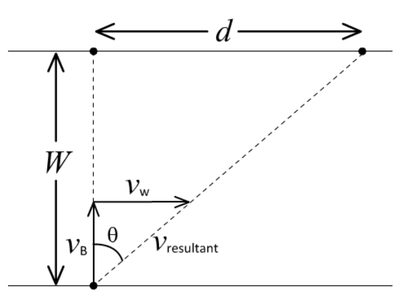
   Consider a boat that can move with a velocity of v_B in still waters. When it crosses a river of width W starting normal to the bank , the minimum time taken by it to cross the river is given by

   $$t = \frac{W}{v_B}$$

   In that interval of time horizontal drift of the boat (due to water) is

   $$d = t \times v_W$$

   $$d = \frac{W}{v_B} \times v_W$$



   Considering the angle formed by the width of the river, and the horizontal drift we get

   $$\tan(\theta) = \frac{d}{W}$$

The above expression indicates that the eventual motion is along the line making an angle of θ w.r.t. the normal to the bank of the river.

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4. Define unit vector, null vector and position vector.

Refer to class notes (or click here for the PDF – page number 06)

5. If $|a+b| = |a-b|$ prove that angle between a and b is 90°.

$|a+b| = \sqrt{a^2 + b^2 + 2ab\cos(\theta)}$ and $|a-b| = \sqrt{a^2 + b^2 - 2ab\cos(\theta)}$

Using the given condition

$|a+b| = |a-b|$

$\sqrt{a^2 + b^2 + 2ab\cos(\theta)} = \sqrt{a^2 + b^2 - 2ab\cos(\theta)}$

$a^2 + b^2 + 2ab\cos(\theta) = a^2 + b^2 - 2ab\cos(\theta)$

$4ab\cos(\theta) = 0$, a and b cannot be zero therefore

$\cos(\theta) = 0$

$\theta = 90^\circ$

6. Show that the trajectory of an object thrown at a certain angle with the horizontal is a parabola.

Refer to class notes (or click here for the PDF – page number 07)

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7. Explain the terms average velocity an instantaneous velocity. When are they equal?

Average velocity is obtained by considering the total displacement of the body in a given interval of time i.e. $\bar{v}_{\text{avg}} = \dfrac{\bar{S}_{\text{total}}}{t_{\text{total}}}$

Instantaneous velocity is obtained by considering the displacement of the body in a very small interval of time i.e. $\bar{v}_{\text{inst}} = \dfrac{d\bar{S}}{dt}$

Average velocity of a body is equal to its instantaneous velocity at all times only if the body is in uniform motion i.e. having constant velocity both in magnitude and direction.

8. Show that the maximum height and range of a projectile are $H = \dfrac{u^2\sin^2(\theta)}{2g}$ and $R = \dfrac{u^2\sin(2\theta)}{g}$ respectively where the terms have their usual meaning.

Refer to class notes (or click here for the PDF – page number 03 and 06)

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9. If the trajectory of a body is parabolic in one reference frame can it be parabolic in another reference fame that moves with a constant velocity with respect to the first reference frame? If the trajectory can be other than parabolic, what else can it be?

Yes the path of projectile may be parabolic in another frame of reference which is moving with constant velocity (v_F).

For example consider a body projected from a moving platform. Such a body may appear to follow a parabolic path both from the moving platform and the ground.

If the horizontal component of the projectile is equal in magnitude and opposite to the direction of the horizontal velocity of the frame of reference then the projectile appears to be executing vertical motion w.r.t. the ground.

If the body is projected vertically w.r.t. the moving platform then it appears to execute a vertical motion w.r.t. platform and parabolic motion w.r.t. the ground. click here for simulation

10. A force $\bar{F} = 2\hat{i} + \hat{j} - \hat{k}$ N acts on a body which is initially at rest. At the end of 20 seconds the velocity of the body is $\bar{v} = 4\hat{i} + 2\hat{j} - 2\hat{k}$ ms^{-1}. What is the mass of the body?

Using the relation

$\bar{F} = m\dfrac{\bar{v} - \bar{u}}{t}$ and considering the initial velocity as zero we get

$$\bar{F} = m\frac{\bar{v}}{t}$$

$$m = \frac{|F|\,t}{|v|} \qquad \text{--- (i)}$$

Magnitude of force is

$$|F| = \sqrt{2^2 + 1^2 + (-1)^2}$$

$$|F| = \sqrt{6}$$

Magnitude of velocity is

$$|v| = \sqrt{4^2 + 2^2 + (-2)^2}$$

$$|v| = \sqrt{24}$$

Substituting values of | F |, | v | in eq (i) and using t = 20 seconds (given)we get

m = 10 kg